Sample Problem 8-2

A 2.0 kg sloth hangs 5.0 m above the ground (Fig. 8-6).

(a) What is the gravitational potential energy U of the sloth–Earth system if we take the reference point $y = 0$ to be (1) at the ground, (2) at a balcony floor that is 3.0 m above the ground, (3) at the limb, and (4) 1.0 m above the limb? Take the gravitational potential energy to be zero at $y = 0$.

KEY IDEA Once we have chosen the reference point for $y = 0$, we can calculate the gravitational potential energy U of the system *relative to that reference point* with Eq. 8-9.

Calculations: For choice (1) the sloth is at $y = 5.0$ m, and

$$U = mgy = (2.0\text{ kg})(9.8\text{ m/s}^2)(5.0\text{ m})$$
$$= 98\text{ J}. \quad \text{(Answer)}$$

For the other choices, the values of U are

(2) $U = mgy = mg(2.0\text{ m}) = 39\text{ J}$,

(3) $U = mgy = mg(0) = 0\text{ J}$,

(4) $U = mgy = mg(-1.0\text{ m})$
$= -19.6\text{ J} \approx -20\text{ J}$. (Answer)

(b) The sloth drops to the ground. For each choice of reference point, what is the change ΔU in the potential energy of the sloth–Earth system due to the fall?

KEY IDEA The *change* in potential energy does not depend on the choice of the reference point for $y = 0$; instead, it depends on the change in height Δy.

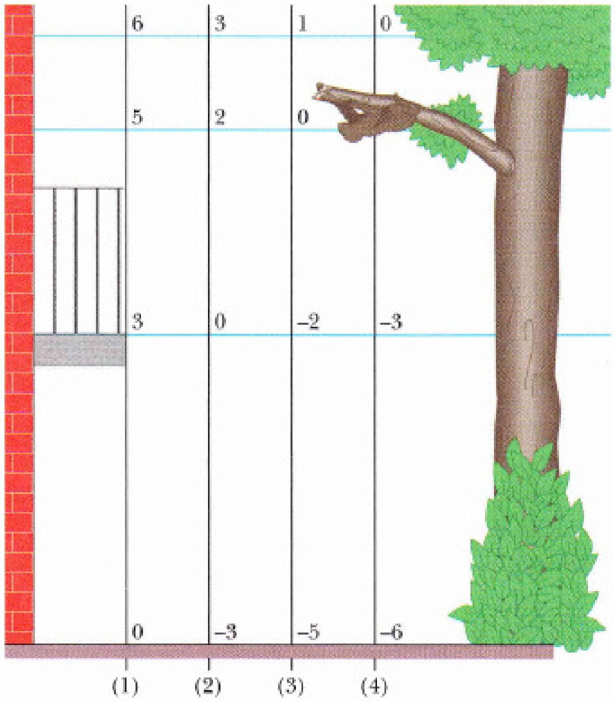

FIG. 8-6 Four choices of reference point $y = 0$. Each y axis is marked in units of meters. The choice affects the value of the potential energy U of the sloth–Earth system. However, it does not affect the change ΔU in potential energy of the system if the sloth moves by, say, falling.

Calculation: For all four situations, we have the same $\Delta y = -5.0$ m. Thus, for (1) to (4), Eq. 8-7 tells us that

$$\Delta U = mg\,\Delta y = (2.0\text{ kg})(9.8\text{ m/s}^2)(-5.0\text{ m})$$
$$= -98\text{ J}. \quad \text{(Answer)}$$

Sample Problem 8-3 Build your skill

In Fig. 8-8, a child of mass m is released from rest at the top of a water slide, at height $h = 8.5$ m above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.

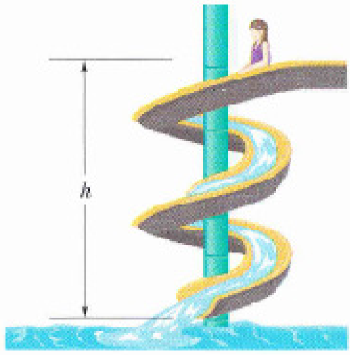

FIG. 8-8 A child slides down a water slide as she descends a height h.

KEY IDEAS (1) We cannot find her speed at the bottom by using her acceleration along the slide as we might have in earlier chapters because we do not know the slope (angle) of the slide. However, because that speed is related to her kinetic energy, perhaps we can use the principle of conservation of mechanical energy to get the speed. Then we would not need to know the slope. (2) Mechanical energy is conserved in a system *if* the system is isolated and *if* only conservative forces cause energy transfers within it. Let's check.

Forces: Two forces act on the child. The *gravitational force*, a conservative force, does work on her. The *normal force* on her from the slide does no work because its direction at any point during the descent is always perpendicular to the direction in which the child moves.

System: Because the only force doing work on the child is the gravitational force, we choose the child–Earth system as our system, which we can take to be isolated.

Thus, we have only a conservative force doing work in an isolated system, so we *can* use the principle of conservation of mechanical energy.

Calculations: Let the mechanical energy be $E_{mec,t}$ when the child is at the top of the slide and $E_{mec,b}$ when she is at the bottom. Then the conservation principle tells us

$$E_{mec,b} = E_{mec,t}. \tag{8-19}$$

To show both kinds of mechanical energy, we have

$$K_b + U_b = K_t + U_t, \tag{8-20}$$

or

$$\tfrac{1}{2}mv_b^2 + mgy_b = \tfrac{1}{2}mv_t^2 + mgy_t.$$

Dividing by m and rearranging yield

$$v_b^2 = v_t^2 + 2g(y_t - y_b).$$

Putting $v_t = 0$ and $y_t - y_b = h$ leads to

$$v_b = \sqrt{2gh} = \sqrt{(2)(9.8\ \text{m/s}^2)(8.5\ \text{m})}$$
$$= 13\ \text{m/s}. \qquad \text{(Answer)}$$

This is the same speed that the child would reach if she fell 8.5 m vertically. On an actual slide, some frictional forces would act and the child would not be moving quite so fast.

Comments: Although this problem is hard to solve directly with Newton's laws, using conservation of mechanical energy makes the solution much easier. However, if we were asked to find the time taken for the child to reach the bottom of the slide, energy methods would be of no use; we would need to know the shape of the slide, and we would have a difficult problem.

Sample Problem 8-6

A food shipper pushes a wood crate of cabbage heads (total mass $m = 14$ kg) across a concrete floor with a constant horizontal force $\vec{F}$ of magnitude 40 N. In a straight-line displacement of magnitude $d = 0.50$ m, the speed of the crate decreases from $v_0 = 0.60$ m/s to $v = 0.20$ m/s.

(a) How much work is done by force $\vec{F}$, and on what system does it do the work?

KEY IDEA Because the applied force $\vec{F}$ is constant, we can calculate the work it does by using Eq. 7-7 ($W = Fd\cos\phi$).

Calculation: Substituting given data, including the fact that force $\vec{F}$ and displacement $\vec{d}$ are in the same direction, we find

$$W = Fd\cos\phi = (40\ \text{N})(0.50\ \text{m})\cos 0°$$
$$= 20\ \text{J}. \qquad \text{(Answer)}$$

Reasoning: We can determine the system on which the work is done to see which energies change. Because the crate's speed changes, there is certainly a change ΔK in the crate's kinetic energy. Is there friction between the floor and the crate, and thus a change in thermal energy? Note that $\vec{F}$ and the crate's velocity have the same direction. Thus, if there is no friction, then $\vec{F}$ should be accelerating the crate to a *greater* speed. However, the crate is *slowing*, so there must be friction and a change ΔE_{th} in thermal energy of the crate and the floor. Therefore, the system on which the work is done is the crate–floor system, because both energy changes occur in that system.

(b) What is the increase ΔE_{th} in the thermal energy of the crate and floor?

KEY IDEA We can relate ΔE_{th} to the work W done by $\vec{F}$ with the energy statement of Eq. 8-33 for a system that involves friction:

$$W = \Delta E_{mec} + \Delta E_{th}. \qquad (8\text{-}34)$$

Calculations: We know the value of W from (a). The change ΔE_{mec} in the crate's mechanical energy is just the change in its kinetic energy because no potential energy changes occur, so we have

$$\Delta E_{mec} = \Delta K = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2.$$

Substituting this into Eq. 8-34 and solving for ΔE_{th}, we find

$$\begin{aligned}\Delta E_{th} &= W - (\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2) = W - \tfrac{1}{2}m(v^2 - v_0^2)\\ &= 20\text{ J} - \tfrac{1}{2}(14\text{ kg})[(0.20\text{ m/s})^2 - (0.60\text{ m/s})^2]\\ &= 22.2\text{ J} \approx 22\text{ J}.\end{aligned} \qquad \text{(Answer)}$$

Sample Problem 8-8

Figure 8-19*a* shows the mountain slope and the valley along which a rock avalanche moves. The rocks have a total mass m, fall from a height $y = H$, move a distance d_1 along a slope of angle $\theta = 45°$, and then move a distance d_2 along a flat valley. What is the ratio d_2/H of the runout to the fall height if the coefficient of kinetic friction has the reasonable value of 0.60?

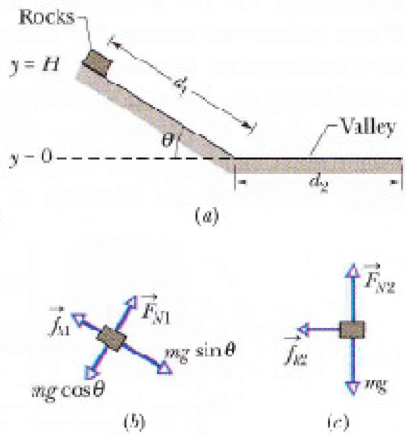

FIG. 8-19 (*a*) The path a rock avalanche takes down a mountainside and across a valley floor. The forces on the rock material along (*b*) the mountainside and (*c*) the valley floor.

KEY IDEAS (1) The mechanical energy E_{mec} of the rocks–Earth system is the sum of the kinetic energy ($K = \frac{1}{2}mv^2$) and the gravitational potential energy ($U = mgy$). (2) The mechanical energy is not conserved during the slide because a (nonconservative) frictional force acts on the rocks, transferring an amount of energy ΔE_{th} to the thermal energy of the rocks and ground. (3) The transferred energy ΔE_{th} is related to the magnitude of the kinetic frictional force and the distance of sliding by Eq. 8-31 ($\Delta E_{th} = f_k d$). (4) The mechanical energy $E_{mec,2}$ at any point during the slide is related to the initial mechanical energy $E_{mec,1}$ and the transferred energy ΔE_{th} by Eq. 8-37, which can be rewritten as $E_{mec,2} = E_{mec,1} - \Delta E_{th}$.

Calculations: The final mechanical energy $E_{mec,2}$ is equal to the initial mechanical energy $E_{mec,1}$ minus the amount ΔE_{th} lost to thermal energy:

$$E_{mec,2} = E_{mec,1} - \Delta E_{th}. \qquad (8\text{-}43)$$

Initially the rocks have potential energy $U = mgH$ and kinetic energy $K = 0$, and so the initial mechanical energy is $E_{mec,1} = mgH$. Finally (when the rocks stop) the rocks have potential energy $U = 0$ and kinetic energy $K = 0$, and so $E_{mec,2} = 0$. The amount of energy transferred to thermal energy is $\Delta E_{th,1} = f_{k1}d_1$ during the slide down the slope and $\Delta E_{th,2} = f_{k2}d_2$ during the runout across the valley. Substituting these expressions into Eq. 8-43, we have

$$0 = mgH - f_{k1}d_1 - f_{k2}d_2. \qquad (8\text{-}44)$$

From Fig. 8-19*a*, we see that $d_1 = H/(\sin\theta)$. To obtain expressions for the kinetic frictional forces, we use Eq. 6-2 ($f_k = \mu_k F_N$). Recall from Chapter 6 that on an inclined plane the normal force offsets the component $mg\cos\theta$ of the gravitational force (Fig. 8-19*b*). Similarly, recall from Chapter 5 that on a horizontal surface the normal force offsets the full magnitude mg of the gravitational force (Fig. 8-19*c*). Substituting these expressions into Eq 8-44 and solving for the ratio d_2/H, we find

$$0 = mgH - \mu_k(mg\cos\theta)\frac{H}{\sin\theta} - \mu_k mgd_2$$

and

$$\frac{d_2}{H} = \left(\frac{1}{\mu_k} - \frac{1}{\tan\theta}\right). \qquad (8\text{-}45)$$