Sample Problem 6-1

If a car's wheels are "locked" (kept from rolling) during emergency braking, the car slides along the road. Ripped-off bits of tire and small melted sections of road form the "skid marks" that reveal that cold-welding occurred during the slide. The record for the longest skid marks on a public road was reportedly set in 1960 by a Jaguar on the M1 highway in England (Fig. 6-3*a*)—the marks were 290 m long! Assuming that $\mu_k = 0.60$ and the car's acceleration was constant during the braking, how fast was the car going when the wheels became locked?

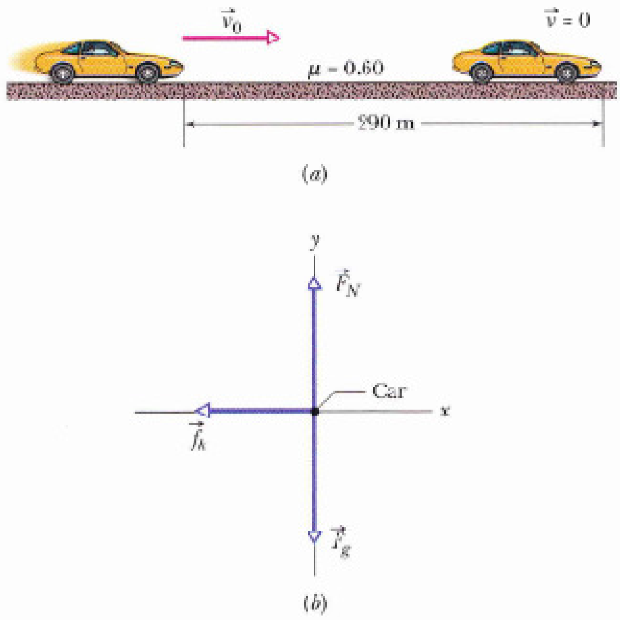

FIG. 6-3 (*a*) A car sliding to the right and finally stopping after a displacement of 290 m. (*b*) A free-body diagram for the car.

KEY IDEAS (1) Because the acceleration a is assumed constant, we can use the constant-acceleration equations of Table 2-1 to find the car's initial speed v_0. (2) If we neglect the effects of the air on the car, acceleration a was due only to a kinetic frictional force $\vec{f}_k$ on the car from the road, directed opposite the direction of the car's motion, assumed to be in the positive direction of an x axis (Fig. 6-3*b*). We can relate this force to the acceleration by writing Newton's second law for x components ($F_{net,x} = ma_x$) as

$$-f_k = ma, \qquad (6\text{-}3)$$

where m is the car's mass. The minus sign indicates the direction of the kinetic frictional force.

Calculations: From Eq. 6-2, the frictional force has the magnitude $f_k = \mu_k F_N$, where F_N is the magnitude of the normal force on the car from the road. Because the car is not accelerating vertically, we know from Fig. 6-3*b* and Newton's second law that the magnitude of $\vec{F}_N$ is equal to the magnitude of the gravitational force $\vec{F}_g$ on the car, which is mg. Thus, $F_N = mg$.

Now solving Eq. 6-3 for a and substituting $f_k = \mu_k F_N = \mu_k mg$ for f_k yield

$$a = -\frac{f_k}{m} = -\frac{\mu_k mg}{m} = -\mu_k g, \qquad (6\text{-}4)$$

where the minus sign indicates that the acceleration is in the negative direction of the x axis, opposite the direction of the velocity. Next, let's use Eq. 2-16,

$$v^2 = v_0^2 + 2a(x - x_0),$$

from the constant-acceleration equations of Chapter 2. We know that the displacement $x - x_0$ was 290 m and assume that the final speed v was 0. Substituting for a from Eq. 6-4 and solving for v_0 give

$$v_0 = \sqrt{2\mu_k g(x - x_0)} = \sqrt{(2)(0.60)(9.8\ \text{m/s}^2)(290\ \text{m})}$$
$$= 58\ \text{m/s} = 210\ \text{km/h}. \quad \text{(Answer)}$$

We assumed that $v = 0$ at the far end of the skid marks. Actually, the marks ended only because the Jaguar left the road after 290 m. So v_0 was at *least* 210 km/h.

Sample Problem 6-2

In Fig. 6-4*a*, a block of mass $m = 3.0$ kg slides along a floor while a force $\vec{F}$ of magnitude 12.0 N is applied to it at an upward angle θ. The coefficient of kinetic friction between the block and the floor is $\mu_k = 0.40$. We can vary θ from 0 to 90° (the block remains on the floor). What θ gives the maximum value of the block's acceleration magnitude a?

normal force is upward, the gravitational force $\vec{F}_g$ with magnitude mg is downward, and (note) the vertical component F_y of the applied force is upward. That component is shown in Fig. 6-4*c*, where we can see that $F_y = F \sin\theta$. We can write Newton's second law ($\vec{F}_{net} = m\vec{a}$) for those forces along the y axis as

$$F_N + F\sin\theta - mg = m(0), \qquad (6\text{-}5)$$

KEY IDEAS Because the block is moving, a *kinetic* frictional force acts on it. The magnitude is given by Eq. 6-2 ($f_k = \mu_k F_N$, where F_N is the normal force). The direction is opposite the motion (the friction opposes the sliding).

Calculating F_N: Because we need the magnitude f_k of the frictional force, we first must calculate the magnitude F_N of the normal force. Figure 6-4*b* is a free-body diagram showing the forces along the vertical y axis. The

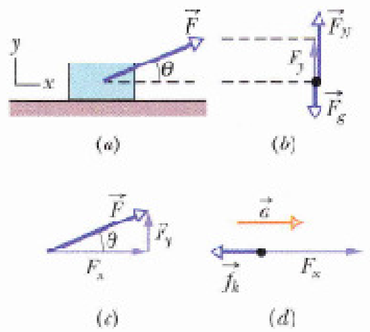

FIG. 6-4 (*a*) A force is applied to a moving block. (*b*) The vertical forces. (*c*) The components of the applied force. (*d*) The horizontal forces and acceleration.

Rearranging and using the identity $(\sin\theta)/(\cos\theta) = \tan\theta$ give us

$$\tan\theta = \mu_k.$$

Solving for θ and substituting the given $\mu_k = 0.40$, we find that the acceleration will be maximum if

$$\theta = \tan^{-1}\mu_k$$
$$= 21.8° \approx 22°. \qquad \text{(Answer)}$$

where we substituted zero for the acceleration along the y axis (the block does not even move along that axis). Thus,

$$F_N = mg - F\sin\theta. \qquad (6\text{-}6)$$

Calculating acceleration a: Figure 6-4*d* is a free-body diagram for motion along the x axis. The horizontal component F_x of the applied force is rightward; from Fig. 6-4*c*, we see that $F_x = F\cos\theta$. The frictional force has magnitude f_k ($= \mu_k F_N$) and is leftward. Writing Newton's second law for motion along the x axis gives us

$$F\cos\theta - \mu_k F_N = ma. \qquad (6\text{-}7)$$

Substituting for F_N from Eq. 6-6 and solving for a lead to

$$a = \frac{F}{m}\cos\theta - \mu_k\left(g - \frac{F}{m}\sin\theta\right). \qquad (6\text{-}8)$$

Finding a maximum: To find the value of θ that maximizes a, we take the derivative of a with respect to θ and set the result equal to zero:

$$\frac{da}{d\theta} = -\frac{F}{m}\sin\theta + \mu_k\frac{F}{m}\cos\theta = 0.$$

Comment: As we increase θ from 0, more of the applied force $\vec{F}$ is upward, relieving the normal force. The decrease in the normal force causes a decrease in the frictional force, which opposes the block's motion. Thus, the block's acceleration tends to increase. However, the increase in θ also decreases the horizontal component of $\vec{F}$, and so the block's acceleration tends to decrease. These opposing tendencies produce a maximum acceleration at $\theta = 22°$.

Sample Problem 6-3

Although many ingenious schemes have been attributed to the building of the Great Pyramid, the stone blocks were probably hauled up the side of the pyramid by men pulling on ropes. Figure 6-5*a* represents a 2000 kg stone block in the process of being pulled up the finished (smooth) side of the Great Pyramid, which forms a plane inclined at angle $\theta = 52°$. The block is secured to a wood sled and is pulled by multiple ropes (only one is shown). The sled's track is lubricated with water to decrease the coefficient of static friction to 0.40. Assume negligible friction at the (lubricated) point where the ropes pass over the edge at the top of the side. If each man on top of the pyramid pulls with a (reasonable) force of 686 N, how many men are needed to put the block on the verge of moving?

second law becomes

$$F_N - mg\cos\theta = m(0). \qquad (6\text{-}11)$$

Solving Eq. 6-11 for F_N and substituting the result into Eq. 6-10, we have

$$f_s = \mu_s mg\cos\theta. \qquad (6\text{-}12)$$

Substituting this expression into Eq. 6-9 and solving for F lead to

$$F = \mu_s mg\cos\theta + mg\sin\theta. \qquad (6\text{-}13)$$

Substituting $m = 2000$ kg, $\theta = 52°$, and $\mu_s = 0.40$, we find that the force required to put the stone block on the verge of moving is 2.027×10^4 N. Dividing this by the assumed pulling force of 686 N from each man, we find that the required number of men is

KEY IDEAS (1) Because the block is on the *verge* of moving, the static frictional force must be at its maximum possible value; that is, $f_s = f_{s,\max}$. (2) Because the block is on the verge of moving *up* the plane, the frictional force must be *down* the plane (to oppose the pending motion). (3) From Sample Problem 5-5, we know that the component of the gravitational force down the plane is $mg \sin\theta$ and the component perpendicular to (and inward from) the plane is $mg \cos\theta$ (Fig. 6-5*b*).

Calculations: Figure 6-5*c* is a free-body diagram for the block, showing the force $\vec{F}$ applied by the ropes, the static frictional force $\vec{f}_s$, and the two components of the gravitational force. We can write Newton's second law ($\vec{F}_{\text{net}} = m\vec{a}$) for forces along the x axis as

$$F - mg\sin\theta - f_s = m(0). \qquad (6\text{-}9)$$

Because the block is on the verge of sliding and the frictional force is at the maximum possible value $f_{s,\max}$, we use Eq. 6-1 to replace f_s with $\mu_s F_N$:

$$f_s = f_{s,\max} = \mu_s F_N. \qquad (6\text{-}10)$$

From Figure 6-5*c*, we see that along the y axis Newton's

$$N = \frac{2.027 \times 10^4\ \text{N}}{686\ \text{N}} = 29.5 \approx 30 \text{ men.} \qquad \text{(Answer)}$$

Comment: Once the stone block began to move, the friction was kinetic friction and the coefficient was about 0.20. You can show that the required number of men was then 26 or 27. Thus, the huge stone blocks of the Great Pyramid could be pulled up into position by reasonably small teams of men.

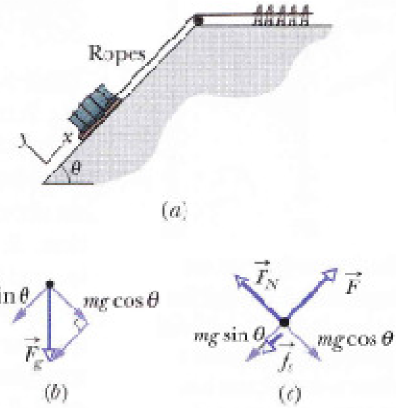

FIG. 6-5 (*a*) A stone block on the verge of being pulled up the side of the Great Pyramid. (*b*) The components of the gravitational force. (*c*) A free-body diagram for the block.

Sample Problem 6-9 **Build your skill**

Upside-down racing: A modern race car is designed so that the passing air pushes down on it, allowing the car to travel much faster through a flat turn in a Grand Prix without friction failing. This downward push is called *negative lift*. Can a race car have so much negative lift that it could be driven upside down on a long ceiling, as done fictionally by a sedan in the first *Men in Black* movie?

Figure 6-12*a* represents a Grand Prix race car of mass $m = 600$ kg as it travels on a flat track in a circular arc of radius $R = 100$ m. Because of the shape of the car and the wings on it, the passing air exerts a negative lift $\vec{F}_L$ downward on the car. The coefficient of static friction between the tires and the track is 0.75. (Assume that the forces on the four tires are identical.)

(a) If the car is on the verge of sliding out of the turn when its speed is 28.6 m/s, what is the magnitude of $\vec{F}_L$?

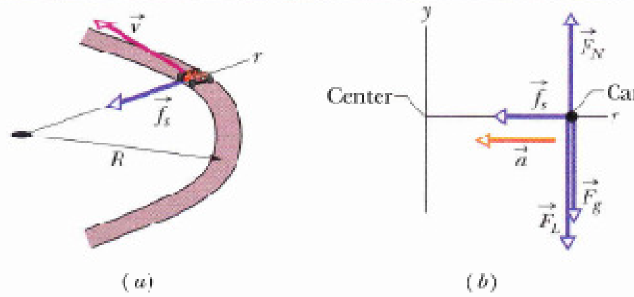

FIG. 6-12 (*a*) A race car moves around a flat curved track at constant speed v. The frictional force $\vec{f}_s$ provides the necessary centripetal force along a radial axis r. (*b*) A free-body diagram (not to scale) for the car, in the vertical plane containing r.

for components along the y axis ($F_{\text{net},y} = ma_y$) as

$$F_N - mg - F_L = 0,$$

or

$$F_N = mg + F_L. \qquad (6\text{-}24)$$

KEY IDEAS

1. A centripetal force must act on the car because the car is moving around a circular arc; that force must be directed toward the center of curvature of the arc (here, that is horizontally).
2. The only horizontal force acting on the car is a frictional force on the tires from the road. So the required centripetal force is a frictional force.
3. Because the car is not sliding, the frictional force must be a *static* frictional force $\vec{f}_s$ (Fig. 6-12*a*).
4. Because the car is on the verge of sliding, the magnitude f_s is equal to the maximum value $f_{s,\max} = \mu_s F_N$, where F_N is the magnitude of the normal force $\vec{F}_N$ acting on the car from the track.

Radial calculations: The frictional force $\vec{f}_s$ is shown in the free-body diagram of Fig. 6-12*b*. It is in the negative direction of a radial axis *r* that always extends from the center of curvature through the car as the car moves. The force produces a centripetal acceleration of magnitude v^2/R. We can relate the force and acceleration by writing Newton's second law for components along the *r* axis ($F_{\text{net},r} = ma_r$) as

$$-f_s = m\left(-\frac{v^2}{R}\right). \tag{6-22}$$

Substituting $f_{s,\max} = \mu_s F_N$ for f_s leads us to

$$\mu_s F_N = m\left(\frac{v^2}{R}\right). \tag{6-23}$$

Vertical calculations: Next, let's consider the vertical forces on the car. The normal force $\vec{F}_N$ is directed up, in the positive direction of the *y* axis in Fig. 6-12*b*. The gravitational force $\vec{F}_g = m\vec{g}$ and the negative lift $\vec{F}_L$ are directed down. The acceleration of the car along the *y* axis is zero. Thus we can write Newton's second law

Combining results: Now we can combine our results along the two axes by substituting Eq. 6-24 for F_N in Eq. 6-23. Doing so and then solving for F_L lead to

$$F_L = m\left(\frac{v^2}{\mu_s R} - g\right)$$

$$= (600 \text{ kg})\left(\frac{(28.6 \text{ m/s})^2}{(0.75)(100 \text{ m})} - 9.8 \text{ m/s}^2\right)$$

$$= 663.7 \text{ N} \approx 660 \text{ N}. \quad \text{(Answer)}$$

(b) The magnitude F_L of the negative lift on a car depends on the square of the car's speed v^2, just as the drag force does (Eq. 6-14). Thus, the negative lift on the car here is greater when the car travels faster, as it does on a straight section of track. What is the magnitude of the negative lift for a speed of 90 m/s?

KEY IDEA F_L is proportional to v^2.

Calculations: Thus we can write a ratio of the negative lift $F_{L,90}$ at $v = 90$ m/s to our result for the negative lift F_L at $v = 28.6$ m/s as

$$\frac{F_{L,90}}{F_L} = \frac{(90 \text{ m/s})^2}{(28.6 \text{ m/s})^2}.$$

Substituting $F_L = 663.7$ N and solving for $F_{L,90}$ give us

$$F_{L,90} = 6572 \text{ N} \approx 6600 \text{ N}. \quad \text{(Answer)}$$

Upside-down racing: The gravitational force is

$$F_g = mg = (600 \text{ kg})(9.8 \text{ m/s}^2)$$

$$= 5880 \text{ N}.$$

With the car upside down, the negative lift is an *upward* force of 6600 N, which exceeds the downward 5880 N. Thus, the car could run on a long ceiling *provided* that it moves at about 90 m/s (= 324 km/h = 201 mi/h).